Control concepts for walking based on point-mass 3D models

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1 Introduction

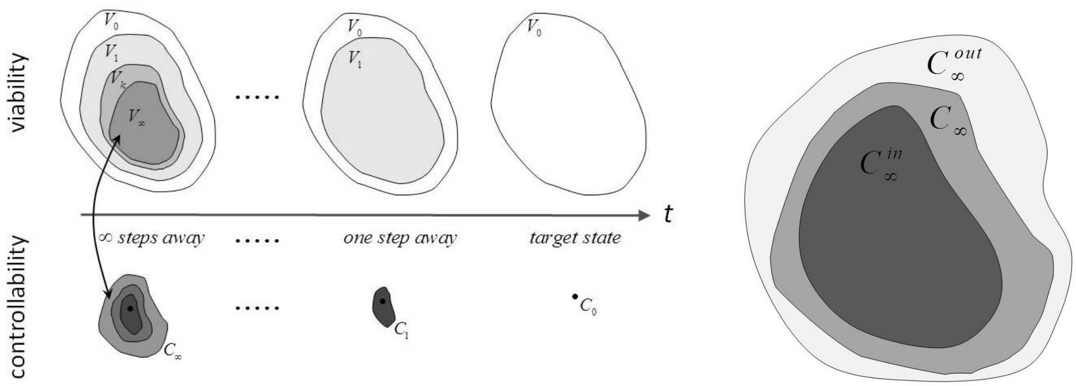
In the field of bipedal locomotion, many researchers use controllers based on simple point mass models to achieve walking balance [1, 2, 3, 4, 5]. In these models, balance is ruled by center of mass (CoM) position and velocity, foot placement and its timing, and push-off impulses. But is this also true for a more complex, many-DOF, robot? Does upper body motion significantly influence CoM dynamics? Most people cannot balance on stilts without taking a step; the Cornell Ranger robot, designed as a biped, can only move its center of mass by about 2cm by swing leg motions. It appears that walking control, even of a multi-DOF robot, is essentially low-dimensional, and that motion of the CoM is efficiently decoupled from the dynamics of the rest of the body. The latter then can be used to pursue other tasks during locomotion, e.g. energy efficiency.

We will justify this idea by considering *viability* and *controllability* concepts for an Inverted Pendulum model (IP, [6, 2]) and a Linear Inverted Pendulum model (LIP, [1]). They can be viewed as generalizations of Pratt's viable and capture regions [4].

2 Viability and Controllability

A given dynamical state of a robot is called 'n-step' viable if there is any way, within the limits of the actuator abilities and any other constraints, for the robot to take at least n steps without falling. By not falling we mean that no part of the robot, except the feet, ever hits the ground and after the next foot placement the robot has a local maximum of potential energy. We define the n-step viable region, V_n, to be the set of all points in the phase space that are n-step viable. That is, V_n represents all the states from which it is possible to take at least n steps without falling. These regions are shown schematically in Fig. 1a. The region V_0 represents the set of all states which are at a local potential energy maximum and for which the robot has not fallen. V_∞ is the limiting region of this sequence - it consists of all points from which it is possible, with appropriate controls, for the robot to never fall.

We define 'controllable' regions with respect to more specific goals. For example we may have a specific location on the ground, or point in phase space, that we desire to achieve. The set of all states satisfying the goal we call C_0. The n-step controllable regions are defined like the n-step viable regions: C_n is the set of all states in phase space that can, with some achievable controls, get to the target region C_0 in n steps or fewer. The controllable region C_∞, the limit of the sequence, is the set of all points from which the biped can always get to a desired state. If the desired state is the upright position with no velocity, then C_n corresponds to Pratt's capture regions [4]. Other examples of



(a) (b)

Figure 1: **Viability and Controllability:** a) schematic of viable and controllable regions; b) comparison of CoM and full model.

a desired state include: the center of mass having a specified speed, the CoM having a desired direction of motion, the foot being on a specified curve on the ground, and the feet landing points being on specified stepping stones (points or small regions on the ground).

For both viable and controllable region definitions we can impose certain constraints that have to be satisfied at each step. These may represent physical constraints of a real robot, or constraints imposed by control goals. Example constraints include: the step time must be at least $t_{\min}$ (swing leg actuator limit), and the feet must fall on a given curve or line on the ground, the feet must fall on given stepping points or in given stepping regions ('stepping stones').

Pratt claims that the viability region V_∞ is close to the controllable region C_∞. We argue that for any bipedal robot these two regions are the same, except possibly for some boundary states, for any target $C_0 \subset V_\infty$ and with the same conditions being applied at intermediate steps for both viability and controllability. That is, being in a state where falling can be avoided indefinitely is equivalent (but for some boundary states) to being able to achieve, eventually, any goal state.

3 Discussion

The concepts of viability and controllability can be applied to particular controller designs. For a specified controller and given goal states the regions then correspond to the basins of attraction of a dynamical system. Based on the sizes of these regions (V_n and C_n), performance of different controllers, as well as specific aspects of a single controller or model, can be compared. For example, we contend that for point mass models (IP or LIP) availability even of a small amount of ankle torque significantly increases sizes of the viable and controllable regions for these models.

We will consider viability and controllability of a few control concepts, including control based on Pratt's capture points and capture regions [4], and control based on Hof's extrapolated center of mass [2]. We plan to create new controller rules, based on trying to get the controlled basins of attraction to fill the viable region V_∞, while still maintaining some simplicity.

Center of Mass Models. We apply the concepts of controllability for more complex bipedal models in order to compare them with IP and LIP models. Let's say we constructed the (∞-step) controllable region for some many-degree-of-freedom model, and C_∞^{out} denotes its projection onto the subspace S_{CoM} defined by CoM position and velocity. This is the outer region in Fig. 1b. It consists of all points in S_{CoM} that are controllable for some higher-dimensional states of the robot. On the same figure, C_∞ is the controllable region for a point-mass model (IP or LIP). Assume all of C_∞ is controlled by some specific controller. If we try to apply the same controller to the complex model, it may or may not work depending on higher-dimensional states. Let's call C_∞^{in} the set of all points in S_{CoM} for which the model is successfully controlled by this controller for all possible higher-dimensional states. This is the most inner region in Fig. 1b. With these definitions in mind we make the following statement:

> Given equivalent step constraints and actuator limits, all three regions (C_∞^{out}, C_∞ and C_∞^{in}) are close to each other.

In the case of C_∞ and C_∞^{in}, this suggests that if a controller works for a point-mass model, than in most cases it will also work for a complex model. Since any point in the region C_∞^{in} is controlled for any higher-dimensional states, only information about the CoM is required for the controller. Therefore, C_∞^{out} being close to the other two regions says that center of mass information is sufficient to control the robot in most cases.

4 Methods

Our primary approach is massive brute-force simulation. Given, say, C_0 we find C_1 by exhaustive search and cell to cell mapping. Then we work progressively back to C_2, etc. Analytic methods and insights will be used to reduce this computational work whenever possible.

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