Quantification of Viability in Simple Models

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1 Motivation

Legged passive dynamic systems that can exhibit a periodic gait that is inherently stable, provide highly desirable templates for the creation of robust legged robotic systems. However, even if a stable limit cycle exists, the system will be unable to recover if a disturbance is large enough. Hence it would be useful to know exactly which states evolve towards the limit cycle and which states lead to failure. The set of all states that evolve towards a certain limit cycle is that cycle's basin of attraction. For a passive system, this basin of attraction is identical to the system's viability kernel; the set of all states that don't lead to inevitable failure. The basin of attraction can be used for the identification of acceptable disturbances and to determine stability beyond a linear first-order approximation.

Knowledge of the basin of attraction is also very valuable for efficient control of legged systems. Imagine a system in a state far from the limit cycle, but in a state that evolves towards this limit cycle. It is then, in principle, not necessary for a controller to perform any work to bring the system closer to the limit cycle. More generally, it will be possible to identify an energetically minimal control intervention that brings the state of the system only back into the basin of attraction, and thus fully reject a disturbance.

2 State of the Art

Due to the strong non-linearity in the studied models, an analytical solution for the basin of attraction is not attainable. A numerical approach consists of repeatedly simulating the system over the whole state space. These numerical calculations show that even for very simple models, basins of attractions can have complicated, fractal-like structures [1]. While the basin of attraction is a good measure of disturbance rejection, it is also computationally expensive to obtain [2]. Although determination of the full basin of attraction is often forgone in lieu of more local assessments of stability, it has been recognized that maximizing the stability of the system does not necessarily maximize the basin of attraction [3].

3 Own Approach

Our approach uses the Poincaré map to construct a first-order return map of the system. The state space is sampled as a grid and each point is mapped to the corresponding state after one step. The numerical simulations are carried out in a MATLAB simulation framework [4]. These simulations are interpolated to obtain a continuous return map. The characterization of failed states is system dependent and somewhat open. Our goal is to have specific rather than general descriptions of failure. As an example, instead of defining failure as the body of the mechanism coming into contact with the ground, we recognize conditions such as tripping or losing all forward momentum as being failed states. Failed states share the characteristic that it must be impossible for a system in a failed state to evolve to a state that is not classified as failure. After identifying the failed states and fixed points on the return map, it is possible to iteratively backtrack in time. As such all initial states that end up in a failed state can be identified, as illustrated in Figure 1. Although the simulation never runs for more than one step, it is possible to predict the behavior after a large number of steps by reasoning on the first-order return map.

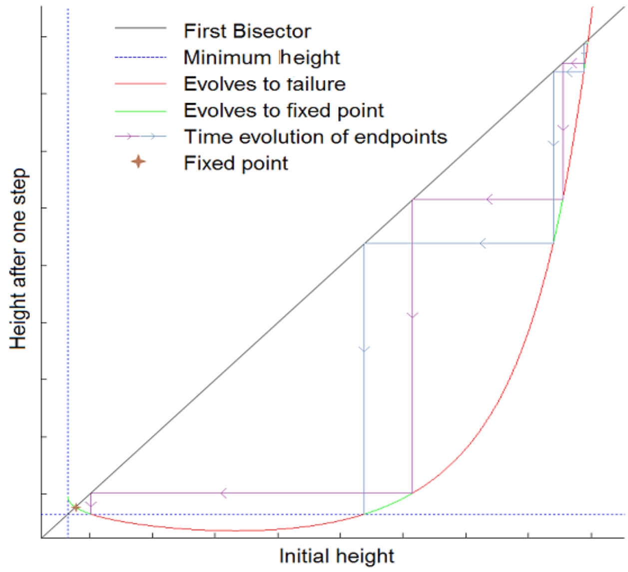

Figure 1: A sample return map. In this example, the intervals of initial conditions that lead to failure by tripping (minimum height is not achieved) are tracked backwards in time. The forwards time evolution of the endpoints of these intervals is shown.

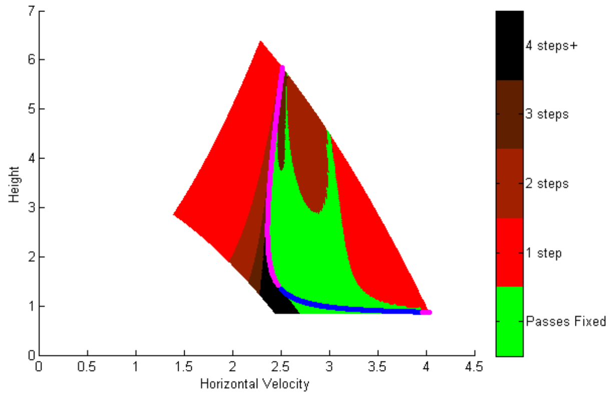

Figure 2: The number of steps to failure for the SLIP-model. Note that the model is energy conserving and thus can only evolve along parabolic trajectories. The green areas converge to a stable fixed point (blue line). In the case that there is no stable fixed point at the current energy level (unstable fixed points are shown in magenta), the green areas indicate states that will only fail after many steps.

4 Current Results

With the above approach we have completely analyzed the SLIP model. It currently takes less than three hours on a personal computer to obtain a detailed description of the basin of attraction for a fixed angle of attack and spring stiffness, an example of which is shown in Figure 2. We can identify all fixed points and their stability. We also can calculate the number of steps to failure and the corresponding failure mode for each initial state. Interestingly, for certain parameters, stable and unstable initial conditions are mixed in a fractal structure. In these regions, a small disturbance can easily put the system from a state that converges to the limit cycle into a state that will fail in a couple of steps, and vice versa. If the energy in the system is raised sufficiently, the stable fixed point becomes unstable. Correspondingly, the limit cycle and the basin of attraction disappear. Even after this point the fractal structure continues to exist, although it contains now states that fail in few steps and states that will fail only after many steps.

5 Best Possible Outcome

We currently aim to expand the model by adding joints and including limitations such as swing times and legs with mass. These limitations would give rise to new failure modes. Additionally the proposed method can also be used to study the viability of active models. Successfully determination of viability of such an active model would allow for the comparison of different controllers.

References

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