A Passive Dynamic Model Predicts All Gaits of Horses

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1 Motivation

Starting in 1965, M. Hildebrand developed criteria to classify quadrupedal gaits by two numbers, the lag of footfall of ipsilateral feet and the duty factor. He compared over 150 different species of animals using this method [1, 2]. The gaits were categorized into two groups: symmetrical gaits and asymmetrical gaits, and analyzed separately. This method allows a graphical classification of gaits in a so-called *gait graph*, and Hildebrand results suggest that the range of possible gaits in this graph can be represented as a continuum, rather than a set of isolated regions. For horses, for example, the regions in the gait graph form a three pronged figure. While certain components of this shape, have been explained [3] , the reasons for its exact shape remain unclear. Why do animals use only certain footfall patterns and avoid others? Moreover, does there exist a gradual transition from one gait to another or are different gaits just overlapping regions in the gait graph.

We seek to answer these questions by using a model-based approach that accounts for the dynamic behavior of quadrupedal locomotion. In our previous work on passive dynamic locomotion with quadrupeds [4, 5], we have shown that a simple model of a conceptual quadruped with elastic legs (Fig. 1) can produce a large variety of different motions. By setting appropriate initial states and model parameters, we were able to create a large number of gaits that are found in nature; including walking, trotting, pacing, toelting, bounding, and galloping. Our model also indicates that these passive dynamic gaits are not isolated samples but that in fact continuous regions of possible gaits exists. Extending on the gaits that we have identified and by continually changing the initial states and model parameters, a series of periodic solutions was found.

2 State of the Art

In 1965, Hildebrand proposed a method to classify symmetrical and asymmetrical quadrupedal gaits separately, only based on the footfall pattern. This became a classical method for quadrupedal gait analysis. A new approach using linear discriminant analysis for both quadrupedal symmetrical and asymmetrical quadrupedal gaits was addressed by Robilliard et al. [6]. In their study, the authors challenged the hypothesis that all gaits can be represented as a continuum and stated that each gait occupies one of multiple overlapping clusters.

To analyze the dynamics of locomotion, people usually use different models to describe the behavior of different gaits. For humans, for example, an inverted pendulum was used to model the dynamics of walking and a spring loaded inverted pendulum was used to model the dynamics of running. In 2006 Geyer et al. have shown that a single spring mass model is sufficient to explain the ground reaction forces of both, human walking and running [7]. Inspired by their work, we have recently shown, that a single compliant leg model can predict all characteristic gaits used by quadrupeds [5].

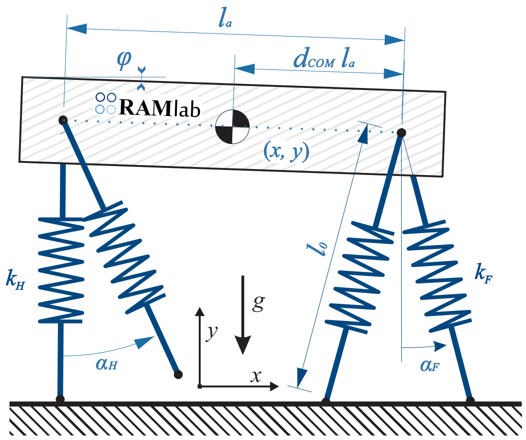

Figure 1: A conceptual quadrupedal model with an extended main body and four massless elastic legs is used to predict the wide variety of quadrupedal gaits found in nature.

3 Own Approach

We developed a simple parameterized model of a passive dynamic quadruped (Fig 1). It consists of a single main body with distributed mass and four massless elastic legs, similar to the bipedal Spring Loaded Inverted Pendulum (SLIP) model. With this model, we identified periodic motions in a MATLAB simulation framework for gait creation [5, 8]. To define a single gait, we introduced two vectors: The vector **X** contains all initial states and the vector **p** all system parameters. Apart from the horizontal position x which is not periodic, **X** includes all continuous states and all continues velocities. Furthermore, it contains a set of discrete states that describe the phase of each leg and the location of contact points on the ground. To obtain a well-defined sequence of ground contact, we defined three distinct leg phases: *stance*, *swing*, and *wait for touch down*. Since a leg cannot make contact during *swing*, modifying the duration of this phase allowed us to prevent feet from striking the ground prematurely. With this, the order of ground contact, which is the main characteriza-

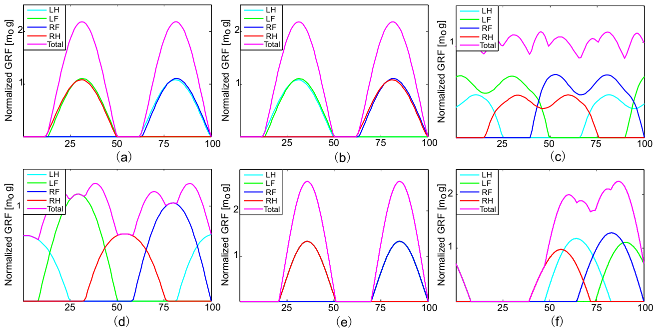

Figure 2: Shown are the vertical ground reaction forces of six different gaits that were identified for a single quadrupedal model: (a) Trotting, (b) Pacing, (c) Walking, (d) Toelting, (e) Pacing, and (f) Galloping.

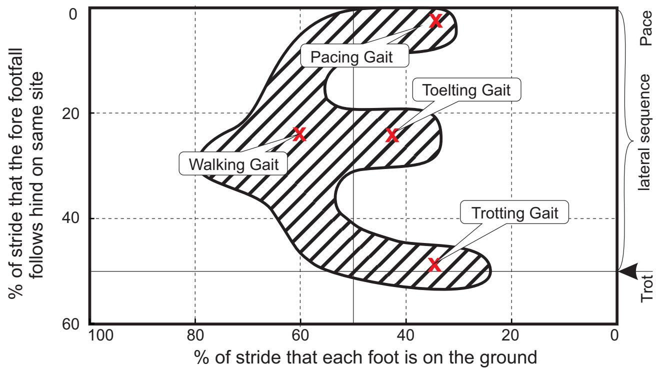

Figure 3: Four symmetrical passive dynamic gaits that were identified with our concepuatp model are compared to the range of gaits found in horses.

tion of a gait, was purely an outcome of the initial states and parameters. The contact sequence was not enforced through additional constraints.

Via numerical integration, a stride-to-stride mapping $\mathbf{X}^{k+1} = \mathbf{P}\left(\mathbf{X}^k, \mathbf{p}\right)$ was established, that maps the initial states $\mathbf{X}^k$ at the beginning of a stride to the values $\mathbf{X}^{k+1}$ at the end of that stride. This stride-to-stride mapping reduces the definition of a periodic gait to the implicit equation: $\mathbf{P}\left(\mathbf{X}^\star, \mathbf{p}^\star\right) - \mathbf{X}^\star \stackrel{!}{=} 0$. Gaits were identified in a single shooting implementation, such that the contact sequence had not to be defined a-priori. The first order stability of a gait was assessed via the eigenvalues of the Monodromy Matrix which is the partial derivative of $\mathbf{P}$ with respect to the continuous and discrete states.

Starting from a set of exemplary gaits found for this simplistic model, additional periodic solutions were identified by gradually changing the initial states and system parameters. Based on the footfall timing of these gaits, the corresponding locations in the gait graph is determined. By varying all variables of the system within reasonable limits and by monitoring first order stability, we can explore the maximum region that our identified gaits can occupy in the gait graph.

4 Current Results

By comparing the model predicted vertical ground reaction forces with those recorded of real horses, we have identified examples for the most important quadrupedal gaits; including walking, trotting, and toelting. Furthermore, we were able to identify two asymmetrical gaits: bounding and galloping. The symmetrical gaits that we obtained are compared to Hildebrand's data in the gait graph of Fig. 3. By gradually varying the initial states and system parameters, our model was able to produce a series of periodic gaits that span larger regions in the gait graph.

5 Best Possible Outcome

Extending on our current results, we will systematically identify the full range of possible passive dynamic quadrupedal gaits. To this end, we will vary system parameters within a reasonable range and monitor first order gait stability. Our expectation is that, within the limitations of our simple planar model, our results will explain some characteristics of the gait graph and on gait selection in nature. Based on the regions that the identified gaits cover in the gait graph, we will investigate how animals select gaits, how they transit from one gait to the other, and whether there exist a gradual change in the gait transition. Furthermore, it is possible, that additional, artificial gaits might be identified that are not fund in nature. These gaits might prove to be beneficial for technical applications.

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