Influence of Uncertainty in Metabolic Dynamic Time Constant on Instantaneous Cost Mapping Techniques

Kimberly Ingraham, Daniel Ferris, and C. David Remy
University of Michigan, Ann Arbor, MI
kaingr@umich.edu

Introduction

Newly developed 'body-in-the-loop' optimization algorithms have the potential to dramatically improve the performance of robotic assistive devices, such as prostheses and exoskeletons. These algorithms minimize a physiological cost function (e.g., energy expenditure) over a range of parameter values (e.g., controller timing) to determine the optimal parameter setting [1]. Successful implementation of these algorithms depends on the underlying instantaneous metabolic cost (i.e., how metabolic expenditure varies as a function of parameter setting). To this end, the relationship between instantaneous energetic cost, x, and experimentally collected breath measurements, y, can be modeled as a first-order linear system with a single subject-specific time constant, τ, according to [1, 3]:

$$y_i = \left(1 - \frac{h_i}{\tau}\right) y_{(i-1)} + \frac{h_i}{\tau} x_i. \tag{1}$$

To implement real-time optimization algorithms, we are interested in the relationship between between instantaneous energetic cost, x, and parameter, p (i.e., the cost landscape, $x(p)$). More importantly, we are interested in the *minimum* of $x(p)$, which corresponds to the energetically optimal parameter value. To obtain an estimate of $x(p)$, one could simply invert (1) and solve for x at each parameter value, using experimental measurements of metabolic cost. Another proposed method expresses $x(p)$ as a polynomial function [1]. The optimal coefficients of this polynomial are determined by computing a pseudo-inverse of a specially-formulated matrix, $\mathbf{A}$, which incorporates the recursive dynamics from (1) and the polynomial function, $x(p)$ [1].

Irrespective of the chosen method to estimate $x(p)$, the identification of the energetically optimal parameter setting depends explicitly on the time constant, τ, of the subject's respiratory dynamics. One common way to identify an individual subject's τ is to induce an instantaneous step change in workload (e.g., increase walking speed from 1.0 m/s to 1.5 m/s) and measure the subject's breath-by-breath response. It is then possible to fit a first-order model to the measured data by minimizing the sum of squared error between the model and each breath. The τ of the best-fit model is taken as the subject's respiratory dynamic time constant. There is a significant amount of inter-subject variability in τ values; a previous study reported time constants from 20-60 seconds for able-bodied subjects walking on a treadmill [3].

Any method for estimating the relationship $x(p)$ requires the collection of experimental breath measurements, y_i across a sequence of parameter values, p_i. Therefore, the estimate of $x(p)$ can be influenced by the sequence of parameters tested. One method, instantaneous cost mapping (ICM), measures metabolic expenditure over a continuous sweep of parameters and estimates $x(p)$ from these data. Various parameter sweeping sequences (e.g., a unidirectional ramp [1] or bidirectional ramp [2] across parameters) have been explored for use in ICM algorithms. Gradient descent techniques, which estimate a local metabolic gradient at an initial parameter and step iteratively towards an energetic minimum, have also been explored [1].

As such, the purpose of this study is twofold. First, we investigate how accurately we are able to estimate τ, and what factors (e.g., signal noise) influence our ability to identify τ on a subject-specific basis. Second, we investigate how uncertainty in τ propagates through the system and affects the identification of the subject's energetically optimal parameter setting. For the purposes of this study, we will use the ICM methodology outlined in [1] to estimate $x(p)$. We hypothesize that uncertainty in τ will increase with the amount of signal noise, but that using a bidirectional ramp parameter exploration strategy will mitigate the effect of this uncertainty on idenfitying a minimum value. The results of this study will inform the refinement of current body-in-the-loop optimization methodologies.

Methods and Results

Uncertainty in Identification of τ

We used computer simulation to examine the effects of three factors on the prediction of τ: noise in the metabolic measurements, magnitude of the workload step size, and the actual time constant (τ_{act}). We created metabolic data by simulating breath dynamics according to (1), and adding white Gaussian noise to the signal. We fit a first-order model to the noisy data to estimate τ of the underlying signal (τ_{est}), which was constrained between 5 and 150 seconds. We repeated this simulation 1000 times for each τ_{act} (20-60 sec), metabolic step size (0.18-0.93 W/kg), and standard deviation (SD) of noise added to the signal (0.0-0.5 W/kg). We fit a normal model to the 1000 τ_{est} values, and compared the standard deviation of the models across conditions (Figure 2.1).

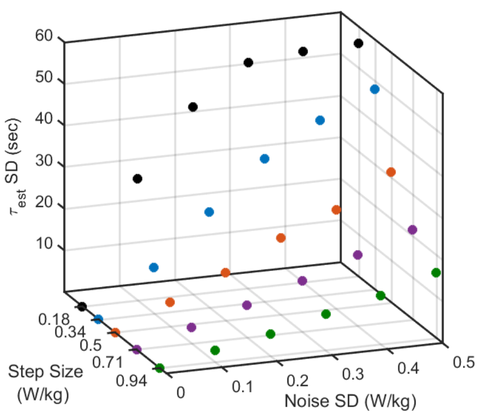

Figure 1: The standard deviation of τ_{est} values (z-axis) increased as signal noise (y-axis) increased and metabolic step size (x-axis) decreased. Results are shown for $\tau_{act} = 40$ sec.

Propagation of Errors in τ

We used computer simulation to investigate how $x(p)$ is affected by errors in τ. We generated "clean" (no noise) metabolic data according to (1). τ_{act} was 45 seconds for these data, and the underlying $x_{act}(p)$ was a parabola, centered at 0. We simulated an ICM protocol that ramped for 8 minutes as either a unidirectional ramp [1] or a bidirectional ramp [2]. We formulated an $\mathbf{A}$ matrix for the same data, but used 10 different τ_{est} values from 10-100 seconds. We then used the pseudo-inverse of the $\mathbf{A}$ matrix to generate each $x_{est}(p)$, and compared the results to $x_{act}(p)$ (Figure 2.2).

Discussion

This study presents preliminary investigations into how uncertainty in estimates of τ affects our ability to identify a minimum of $x(p)$ using ICM techniques. As hypothesized, uncertainty in τ increased as signal noise increased and step size decreased (Figure 2.1). In practice, given some measurable signal noise, these data could be used as

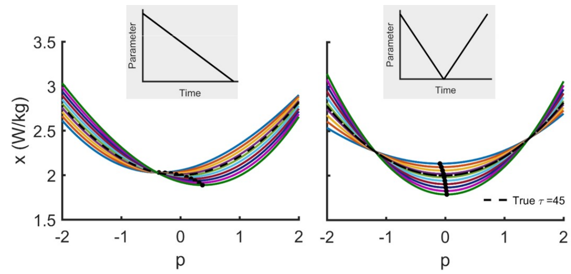

Figure 2: The unidirectional ramp (left) resulted in minimum locations (x-values) within ±10% of the actual minimum; the corresponding metabolic cost (y-values) was between 3.5% higher and 14% lower than the actual minimum. The bidirectional ramp (right) resulted in minimum locations within ±2.5% of the actual minimum; the corresponding metabolic cost was between 26% higher and 17% lower than the actual minimum.

a lookup table to determine how large a step is necessary to obtain a desired confidence in the estimate of τ. Future work in this area will focus on analyzing a variety of statistical distributions to best represent the uncertainty in τ estimates. However, as shown in Figure 2.2, the use of a bidirectional ramp during the ICM protocol mitigates the effect of uncertainty in τ on the estimate of the minimum, compared to a unidirectional ramp. In an experimental setting, noise levels in the metabolic measurements can far exceed those tested in this study, which would further increase uncertainty in the subject's τ value. Therefore, the results of this study suggest that due to the known dynamic delays and noise of respiratory measurements, the use of a bidirectional ramp parameter sweep should be considered best practice for ICM methodology. Future work will focus on deriving analytical expressions to describe how error in estimates of τ propagate through the system. It is not yet clear how close our simulations would match experimental data, so we will also investigate the effects of τ uncertainty during human locomotion with robotic assistive devices.

References

[1] W. Felt et al. "Body-In-The-Loop": Optimizing Device Parameters Using Measures of Instantaneous Energetic Cost. *Plos One*, 10(8):e0135342, 2015.

[2] J. R. Koller et al. "Body-in-the-Loop" optimization of assistive robotic devices: A validation study. *Robotics: Science and Systems Conference*, Submitted, 2016.

[3] J. C. Selinger and J. M. Donelan. Estimating instantaneous energetic cost during non-steady state gait. *Journal of Applied Physiology*, 117:1406–1415, 2014.