Proceedings of the ASME 2007 Summer Bioengineering Conference (SBC2007)
June 20-24, Keystone Resort & Conference Center, Keystone, Colorado, USA

SBC2007-176521

OPTIMAL ESTIMATION OF DYNAMICALLY CONSISTENT KINEMATICS AND KINETICS FOR FORWARD DYNAMIC SIMULATIONS

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INTRODUCTION

Forward dynamic simulation provides a powerful framework for characterizing in vivo loads, for investigating the muscular coordination of movement, and for predicting changes in movement due to injury, impairment or surgical intervention. However, the computational challenge of generating simulations has greatly limited the use and application of dynamic models. Traditional approaches use optimization to determine a set of input trajectories (e.g. muscle forces or joint torques) that drive a model to track experimental motion and force measurements [1,2]. Optimization is needed, in part, to resolve dynamic inconsistencies between measured kinematics and ground reactions. Large scale dynamic optimization problems of this type are inherently difficult to solve, often necessitating model simplifications. It has previously been shown that dynamic inconsistencies can be efficiently resolved on a per-frame basis by enforcing whole-body dynamic constraints [3,4]. However, forward simulations cannot be generated from such data since integration of the accelerations will not re-produce measured velocities and positions.

In this study, we introduce an optimal estimation approach to efficiently solve for generalized accelerations that best agree with measured accelerations and forces at each time step. The estimated accelerations are numerically integrated to enforce dynamic consistency over time. Numerical optimization is then used to determine a set of initial generalized coordinates and speeds that produce a movement most consistent with the measured motion. We show that the proposed residual elimination algorithm (REA) converges to an accurate solution, reduces the detrimental effects of measurement errors on joint torques, and eliminates the need for residual forces that arise in standard inverse dynamics. REA can thus be used as a basis for generating accurate simulations of subject-specific movement dynamics.

METHODS

Residual Elimination

Segment movements (described by a generalized coordinate vector *q*) and ground reactions (Forces *F* and Moments *M*), as they are typically recorded in gait labs, are not fully independent quantities. They are coupled by the Newton-Euler equations of motion (EOM) that impose a dynamic constraint between accelerations and external reactions. The residual forces that arise during inverse kinematics analyses are a consequence of the violation of this constraint by erroneous measurements and/or model inaccuracies.

The six overall Newton-Euler equations of a three-dimensional linked segment system can be stated in the following linear form:

$$A(\vec{q})\begin{bmatrix}\vec{F}\\ \vec{M}\\ \ddot{\vec{q}}\end{bmatrix} = f\left(\vec{q},\dot{\vec{q}}\right) \qquad (1)$$

which necessitates that the external forces and segment accelerations balance the forces due to gyroscopic and gravitational effects. We added small variations ($\delta F, \delta M, \delta\ddot{q}$) to the measured forces and accelerations ($F', M', \ddot{q}'$) that would satisfy (1) and thus enforce dynamic consistency. Variations were mathematically separated from the measurements to obtain a set of underdetermined equations of the form $A\delta = b$. These equations were subsequently solved using the weighted right Moore-Penrose pseudo-inverse, resulting in a solution vector δ that minimized the norm $\|W^{-1}\delta\|$. The weight matrix W was set to a diagonal matrix of estimated measurement standard deviations. Thus, the solution obtained eliminated the residual forces at the base segment while distributing the errors optimally over the entire model in accordance with the precision of the corresponding measurements.

Integration and Initial Conditions

The preceding processing of the kinetic and kinematic data was expressed on a per-frame basis and entirely in terms of accelerations. The Newton-Euler equations return no information about position and velocity. On the contrary, these values are necessary in (1) to evaluate the matrix A, and to determine the current gravitational and gyroscopic forces. The only feasible way of obtaining these values is via numerical integration of the computed generalized accelerations. Unfortunately, the alterations made to the measured accelerations cause the integrated motion to deviate from the recorded trajectories – an effect that is further augmented by the inherently unstable dynamics of gait. To circumvent this, numerical optimization was used to determine a set of initial generalized coordinates and generalized speeds that, after solving (1) for accelerations and numerically integrating, produced a motion that best replicated the recorded motion (Fig. 1). This optimization problem was solved using a non-linear gradient-based routine (sequential quadratic programming) that minimized a cost function defined as a weighted sum of squared differences between measured and model-predicted marker positions.

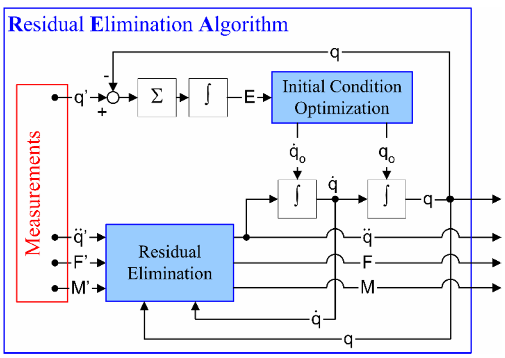

Figure 1. Schematic of the Residual Elimination Algorithm

Evaluation

REA was evaluated using a forward dynamic simulation of a full cycle of gait [5] to artificially create marker and force plate data. The simulation had no residuals and the joint torques were known, thereby providing a ground truth by which to compare the results. Normally distributed white noise, with standard deviations varying from 0 to 20 mm, were added to the artificial marker kinematics in all three spatial directions prior to processing. Inverse kinematics analysis was performed on the noise-contaminated marker data to obtain generalized coordinate trajectories. These trajectories were then low-pass filtered (6 Hz) and spline-fitted. A 3-dimensional, 29-dof full body model was used for simulation and analysis. Deviations in the generalized accelerations and ground reactions were estimated assuming measurement sd's of external forces and moments were negligible, and that measurement sd's of generalized accelerations were equal for all coordinates. Optimization of the initial conditions was performed for the entire set of generalized coordinates and speeds, resulting in 58 optimization variables. Inverse dynamic analysis was then performed with the original and with the processed REA generated kinematics to determine the effect of the processing on the computed joint torques.

RESULTS

When performing standard `bottom up' inverse dynamics analysis [6] on noise-contaminated (sd =12 mm) marker data, residual forces (50±38 N) and torques (14±9 Nm) arose at the base segment (pelvis). The proposed algorithm correctly eliminated these residual forces and created a motion that was dynamically balanced and consistent over time. Despite the huge (58 dimensional) search space and the complex objective function, the numerical optimization was well behaved, and consistently converged to an optimal set of initial conditions in ~100 minutes. The processing also improved the accuracy of the joint torque estimation, by approximately 31% at the hip and by 13% at the knee. Errors remained relatively constant at the ankle. Comparing the noisy marker trajectories with marker positions predicted by REA, the algorithm was able to reduce marker noise by about 9%. These results suggest that the proposed method is a powerful tool to reduce the adverse effects of measurement noise and soft tissue movement. Lowering the number of estimated accelerations to the 6 dof's of the pelvis substantially reduced the computation time (to ~4 minutes) while only slightly degrading the ability to reject noise.

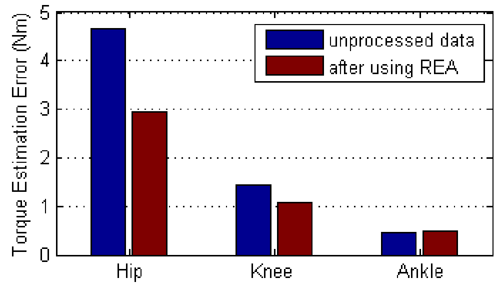

Figure 2. Average torque estimation error for hip, knee, and ankle flexion and extension with 12mm Noise SD

There are three noteworthy aspects of REA. First, the computational feasibility, in part, derives from considering only the 6 overall EOM in (1). While it is feasible to use the entire set of EOM as done by others [3,4], this would also add additional unknowns (joint torques) to the estimation problem which could seriously degrade performance. Secondly, we have applied REA to experimental gait data and obtained similar performance to that described in this abstract. While it is not possible to quantify noise rejection for experimental data, the changes to the generalized coordinates were relatively small (e.g. ~1° deviations for joint angles) suggesting the approach is robust. Finally, the accelerations computed by REA can be numerically integrated to re-create the estimated kinematics, thereby providing a consistent set of data for efficiently generating forward simulations of gait. We conclude that application of the proposed algorithm to subject-specific gait dynamics may facilitate greater use of simulations in clinical studies.

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ACKNOWLEDGEMENTS

NIH AG24276 and Simbios